UNIT-3 Boundary Layer Theory & Dimensional Analysis.

* A Real Fluid Flows as a Solid body (or) Solid ball either the Fluid particles to the Boundary and Condition of No slip occurs the velocity of Fluid and Solid body of to close action. To the boundary will be Same. and the Fluid will be stationary. The action will be zero.

* A very thin layer of fluid of boundary layer the variation from zero at the Solid boundary to Free stream velocity in the direction normal to the Boundary take place. Touch of fluid with Solid body Exerts a Shear Stress on the fall in the direction of Motion.

$$\tau = \mu \cdot \frac{\partial u}{\partial y}$$

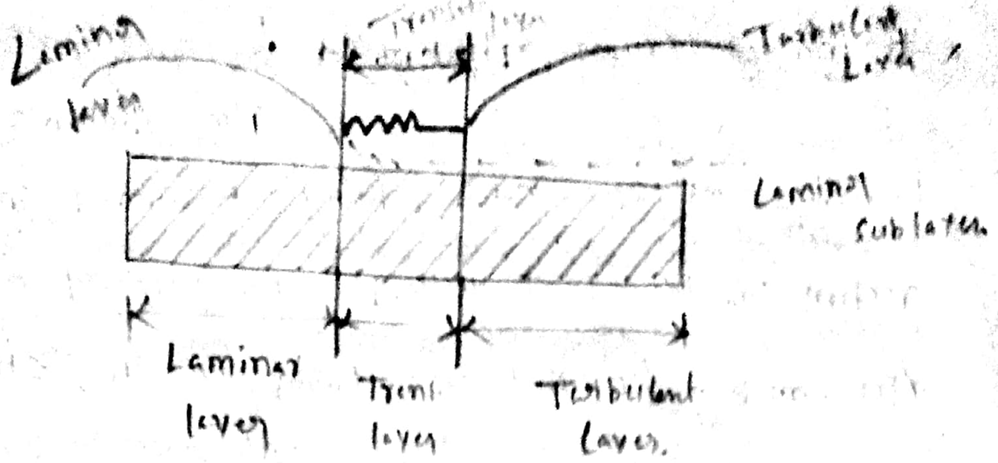
The outside of Boundary layer at velocity is Const or equal to Free stream Position.

Free stream velocity :-

* It is defined as an Area of average absolute Velocity at the inlet

* They are three types of Boundary layer.

1. Laminar Boundary layer
2. Turbulent Boundary layer.
3. Transition Boundary layer



Laminar layer:-

The boundary considers the flow of fluid having free stream velocity over a smooth thin plate which is plat a place parallel to the direction of free stream velocity. That fluid velocity and surface velocity should be equal.

Reynold's Number $Re = < 2000$

Turbulent Layer:-

The length of the plate is more than the distance the thickness of the boundary layer will be go on increase the down stream direction

Reynold's Number $Re = > 4000$

Transition layer:-

The fluid of laminar layer becomes unstable and motion of fluid is distributed and irregular action is called Transition from laminar to turbulent layer.

Laminar sub layer:- The region of Turbulent Boundary layer zone of solid surface level is called Laminar sublayer.

* Boundary Layer Thickness :-

The distance from the Boundary of the solid body measured in the y-direction to the point where the velocity of fluid is approximately equal to 0.99 times the free stream (m) velocity of fluid.

* Displacement thickness:-

The distance measured perpendicular to the boundary of the solid body by which the boundary should be compensate for the Reduction in flow rate on account of boundary layer function.

Momentum thickness:-

The distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the following fluid on account of boundary layer formation.

Energy thickness:-

The distance measured perpendicular to the formed any of the solid body, by which the boundary should be displaced to compensate for reductions in K.E of the following fluid on account of boundary layer formation. {FORMULAS}

30/11/18

Displacement thickness :- $\delta^* = \int_0^\delta \left[1 - \frac{u}{U}\right] \cdot dy$

Momentum thickness:- $\theta = \int_0^\delta \frac{u}{U}\left[1-\frac{u}{V}\right] dy$

Energy thickness:- $\delta_e = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$.

$Re = \frac{\rho V d}{\nu}$; $\nu = \frac{\mu}{\rho}$; $\tau_0 = \mu \cdot \frac{du}{dy}$.

The velocity distribution in the boundary layer is given by $\frac{u}{U} = \frac{y}{\delta}$ where.

u is velocity at a distance from the flame.

$u = U$ & $y = \delta$

where δ being boundary layer thickness. Find

(i) Displacement thickness

(ii) Momentum thickness

(iii) Energy thickness

(iv) Value of $\frac{\text{displacement thickness}}{\text{Momentum thickness}}$.

Sol: Velocity distribution $\frac{u}{U} = \frac{y}{\delta}$

1. $\delta^* = \int_0^\delta \left[1 - \frac{y}{\delta}\right] dy$

$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta$ $\because u = U$, $y = \delta$

Substitute $y = \delta$ in Above Equation.

$= \left[\delta - \frac{\delta^2}{2\delta}\right]_0^\delta$

$\delta^* = \left[\delta - \frac{\delta}{2}\right] = \frac{\delta}{2}$.

2. $\theta = \int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right] dy$

$$= \int_0^{\delta} \frac{y}{\delta}\left[1 - \frac{y}{\delta}\right] dy$$

$$\theta = \frac{\delta}{6}.$$

3. $$\delta e = \int_0^{\delta} \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy$$

$$= \int_0^{\delta} \frac{y}{\delta}\left[1 - \frac{y^2}{\delta^2}\right] dy$$

$$\delta e = \frac{\delta}{4}.$$

The velocity distribution in laminar boundary layer over a flat plate is assumed to be given by second order polynomial $u = a + by + cy^2$ determine it. From using the necessary boundary conditions.

Sol:

$$u = a + by + cy^2$$

Boundary Conditions

$$y = 0, \quad u = 0$$

$$0 = a + b \times 0 + c \times 0^2$$

$$a = 0.$$

$$\boxed{u = by + cy^2} \qquad \begin{Bmatrix} u = U \\ y = \delta \end{Bmatrix}$$

$$\left[\frac{du}{dy}\right]_{y=\delta} = \frac{d}{dy}(by + cy^2)$$

$$0 = b + 2cy$$

$$\boxed{b = -2cy,} \quad \text{"b" Combination}$$

$$u = by + cy^2$$

$$= y(-2cy) + cy^2$$

$y = \delta$

$= \delta(-2c\delta) + c\delta^2$

$= -2c\delta^2 + c\delta^2$

$u = -c\delta^2$

$$\boxed{c = -\frac{u}{\delta^2}}$$ "C" combination

$b = -2c\delta$

$= -2 \times \frac{-u}{\delta^2} \times \delta$

$= \frac{2u}{\delta}$

Now Simplyfying $u = by + cy^2$

$= \frac{2U}{\delta} y + \left(\frac{-U}{\delta^2} y^2\right)$

$\frac{u}{U} = \frac{2y}{\delta} - \frac{y^2}{\delta^2}$

$$\boxed{\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2}$$

By using the Combination of $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$.

Find out the displacement & Momentum & Energy thickness.

Displacement thickness:-

$\delta^* = \int_0^\delta \left(1 - \frac{u}{V}\right) dy$

$= \left[y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta = \delta - \delta + \frac{\delta}{3}$

$= \delta/3$

Momentum thickness:-

$\theta = \int_0^\delta \frac{u}{V}\left[\left(1 - \frac{u}{V}\right)\right] dy$

$= \int_0^\delta \left(\frac{u}{V} - \left(\frac{u}{U}\right)^2\right] dy$

$= \int_0^\delta 2(y/\delta) - (y/\delta)^2 - \left[2(y/\delta) - (y/\delta)^2\right]^2.$

The velocity distribution in the boundary layer $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$

Find the Displacement thickness, Momentum & Energy thickness.

Sol: (i) Displacement Thickness:-

$$\delta^* = \int_0^\delta \left[1-\frac{u}{U}\right]dy \Rightarrow \int_0^\delta \left[1-(y/\delta)^{1/7}\right]dy$$

$$\Rightarrow \delta - \frac{7}{8}\cdot\delta^{1/7} \Rightarrow [y]_0^\delta - \frac{1}{\delta^{1/7}}\left[\frac{y^{8/7}}{8/7}\right]_0^\delta$$

$$\Rightarrow \delta - \frac{\delta^{8/7}}{\delta^{1/7}}\cdot 7/8 \Rightarrow \delta - \delta^{7/7}\cdot 7/8 \Rightarrow \delta/8.$$

(ii) Momentum thickness:-

$$\theta = \int_0^\delta \frac{u}{U}\left[1-u/U\right]dy$$

$$\Rightarrow \int_0^\delta (y/\delta)^{1/7}\left(1-(y/\delta)^{1/7}\right)dy$$

$$\Rightarrow \int_0^\delta \left[(y/\delta)^{1/7} - (y/\delta)^{2/7}\right]dy$$

$$\Rightarrow \frac{1}{\delta^{1/7}}\left(\frac{\delta^{8/7}}{8/7}\right) - \frac{1}{\delta^{2/7}}\left(\frac{\delta^{9/7}}{9/7}\right)$$

$$\Rightarrow \frac{7\delta}{8} - \frac{7\delta}{9}$$

$$\Rightarrow \frac{63\delta - 56\delta}{72}$$

$$\theta = \frac{7\delta}{72}$$

(iii) Energy Thickness

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left[1-\frac{u^2}{U^2}\right]dy$$

$$= \int_0^\delta (y/\delta)^{1/7}\left[1-(y/\delta)^2\right]dy$$

$$= \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7}\left(1-{}^1/_\delta{}^{2/7}\right)dy$$

$$= \int_0^\delta \left(\left(\frac{y}{\delta}\right)^{1/7} - \left(y/\delta\right)^{3/7}\right)dy$$

$$= \frac{\delta^{8/7} \cdot \delta^{-1/7}}{8/7} - \frac{\delta^{10/7}\cdot\delta^{-3/7}}{10/7}$$

$$= \frac{\delta^{7/7}}{8/7} - \frac{\delta^{7/7}}{10/7}$$

$$= 7\delta/8 - \frac{7\delta}{10}$$

$$= \frac{(70-56)\delta}{80} = \frac{14\delta}{80}$$

$$\delta^{**} = 7\delta/40$$

1.8.18 The velocity distribution in a boundary layer over the face of a spill way observed to the small $\frac{u}{U} = (\frac{y}{\delta})^{0.22}$. The free stream velocity $U = 20 m/s$ & boundary thickness is 5cm at a certain section the discharge of spill way $5 m^3/sec$. calculate the displacement thickness, energy thickness & loss of energy up to section under considerations.

Sol: Given data:-

$$\frac{u}{U} = (\frac{y}{\delta})^{0.22} = \int_0^\delta (1 - \frac{u}{U}) dy$$

$$= \int_0^\delta 1 - (y/\delta)^{0.22}) dy$$

$$= \left[y - \frac{y^{1.22}}{1.22 \delta^{0.22}} \right]_0^\delta dy$$

$$= \delta - \frac{\delta^{1.22 - 0.22}}{1.22}$$

$$= \delta - \frac{\delta}{1.22}$$

$$= \delta (1 - \frac{1}{1.22})$$

$$= 0.18\delta$$

$$= 0.18 \times 9 = 0.9$$

loss of energy $E_2 = \frac{1}{2} \rho \delta_e U^3$

$$= \frac{1}{2} \times 1000 \times 1.1 \times 20^3 \times 10^{-3}$$

The Boundary layer thickness at a distance of 1m from the leading edge of a Flat Plate kept over zero angle of incident at flow direction is 1mm. The velocity out side the Boundary layer 25m/sec. The B.L. Thickness at a distance 4m. Find the Boundary layer of a thickness of a section 2

Sol:

$$\delta = \frac{5x}{\sqrt{Re}}$$

$$\delta_1 = \frac{5x}{\sqrt{\frac{Ux_1}{\nu}}}$$

$$\delta_2 = \frac{5x}{\sqrt{\frac{4x_1}{\nu}}}$$

$$\frac{\delta_1}{\delta_2} = \frac{5x / \sqrt{\frac{4x_1}{\nu}}}{5x / \sqrt{\frac{5x_1}{\nu}}}$$

$$\frac{\delta_1}{\delta_2} \Rightarrow {}^1/\delta_2 = \sqrt{\frac{x_2}{x_1}}$$

$$\delta_2 = \sqrt{\frac{x_1}{x_2}}$$

$$\delta_2 = \sqrt{\frac{1}{4}} = 0.5m = 50mm.$$

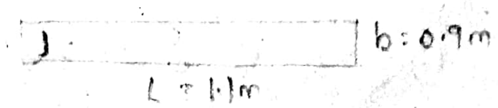
B.L.T

S.No		δ	C_D^*	C_D
1	$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$	$\frac{5.48x}{\sqrt{Re_x}}$	$\frac{0.73}{\sqrt{Re_x}}$	$\frac{1.46}{\sqrt{Re_L}}$
2	$\frac{u}{U} = \frac{3}{2}(y/\delta) - \frac{1}{2}(y/\delta)^3$	$\frac{4.64x}{\sqrt{Re_x}}$	$0.646/\sqrt{Re_x}$	$1.292/\sqrt{Re_L}$
3	$\frac{u}{U} = 2(y/\delta) - 2(y/\delta)^3 + (y/\delta)^4$	$5.84x/\sqrt{Re_x}$	$0.686/\sqrt{Re_x}$	$1.372/\sqrt{Re_L}$
4	$\frac{u}{U} = \sin(\frac{\pi}{2} \cdot y/\delta)$	$4.795x/\sqrt{Re_x}$	$0.654/\sqrt{Re_x}$	$1.31x/\sqrt{Re_L}$
5	Blasius results ($Re < 3.2\times10^5$)	$5x/\sqrt{Re_x}$	$0.664/\sqrt{Re_x}$	$1.328/\sqrt{Re_L}$

Above the table consists of the values of δ (boundary layer thickness), C_D^* (Local co-efficient of drag), C_D (average co-efficient of drag) in terms of Reynolds Number (Re) for various velocity profiles/ distributions.

PROBLEMS ON LAMINAR

2/8/18
1. Air is flow a over a smooth flat plate with a velocity of 12m/s. The velocity profile is $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$. The length of plate 1.1m & width 0.9m. Laminar Boundary layer exist up to a value of $Re = 2\times10^5$ & Kinematic viscosity of Air is 0.15 stroke. Find max. distance & Max. thickness?



Sol: Velocity = 12m/s

length = 1.1m

width = 0.9m

$Re = 2\times10^5$.

$Re = \frac{u \cdot x}{\nu}$

$2\times10^5 = \frac{12\times x}{0.15\times10^{-4}}$

Max. distance $x = 0.25m$ of Air flow.

$\delta = \frac{5.48\, x}{\sqrt{Re_x}}$

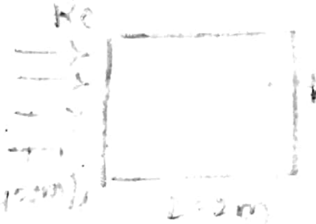
$= \frac{5.48\times0.25}{\sqrt{2\times10^5}}$

$= 0.00306m$

$= 3.06mm.$

2. Oil with a free stream velocity of 2m/sec. flow over thin plate 2m wide & 2m long. Calculate the Boundary layer thickness at the trailing end point & determine total surface resistance of the plate. Take spe. gravity 0.86 & kinematic viscosity 10^{-5} m/sec

Sol: Given:



wide = 2m

long = 2m

Sp. gravity = 0.86

K. viscosity $= 10^{-5}$ m/s

$\delta = ?$

{NOTE: it is the turbulent means 5×10^5 to 10^7

laminar $<5\times10^5$ }

$Re = \frac{U \cdot L}{\nu} = \frac{2\times2}{10^{-5}} = 4\times10^5$

(so it is laminar flow).

$\delta = \frac{5L}{\sqrt{Re_x}} = \frac{5\times2}{\sqrt{4\times10^5}} = 0.015m = 15mm.$

Surface Resistance of the plate $F_D = \frac{1}{2}\rho A u^2 \times C_D$

$C_D = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{4\times10^5}} = 0.00$

$\rho = 1000\times0.86$ kg/m³

Sp. gr = 0.86 = 860 kg/m³

$F_D = \frac{1}{2} \rho A U^2 \times C_D$ { Area of plate $A = L \times b = 2 \times 2 = 4$ }

$= \frac{1}{2} \times 860 \times 4 \times 2^2 \times 0.0020$

$= 13.76$

Total Resistance $T_{FD} = 2 \cdot F_D = 2 \times 13.76 = 27.52$

(Here, 2 means 2 sections like Front side & Back side Both such as 2 sections).

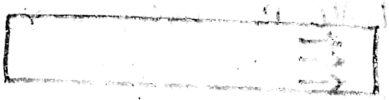
PROBLEMS ON TURBULENT :-

⇒ The Reynolds number is More than 5×10^5 & $< 10^7$ of thickness of Boundary layer.

$$\delta = \frac{0.37 \cdot x}{Re^{1/5}}, \quad C_D = \frac{0.072}{Re^{1/5}}$$

Determine the thickness of the Boundary layer of the trailing edge of smooth plate of length 4m & width 1.5m. when the plate is moving with a velocity of 4m/s in stationary Air. Take kinematic viscosity of Air 1.5×10^{-5} m²/sec.

Sol: First we have to find out



$U = 4 m/s$

$\nu = 1.5 \times 10^{-5} m^2/s$

$$Re = \frac{U \cdot L}{\nu} = \frac{4 \times 4}{1.5 \times 10^{-5}} = 10.6 \times 10^5$$

{ $< 5 \times 10^5$ - Laminar
5×10^5 to 5×10^7 - Turbulent }

According to above Conditions it is a Turbulent flow.

Now Boundary thickness

$$\delta = \frac{0.37 \times L}{Re^{1/5}}$$

$$= \frac{0.37 \times 4}{(10.6 \times 10^5)^{1/5}} = 0.092m = 92mm$$

To Find the Surface distance of a plate the Air. density of Sp.gravity 0.9.

$$F_D = 1/2 \rho A u^2 \times C_D$$

S.G = 0.9

ρ = 900 kg/m³

A = L × b

= 4 × 1.5 = 6

$$= 1/2 \times 900 \times 6 \times 4^2 \times C_D$$

Find Drag Co-efficient $C_D = \frac{0.072}{Re^{1/5}}$

$$= \frac{0.072}{(10.6 \times 10^5)^{1/5}} = 0.004$$

$$= 1/2 \times 900 \times 6 \times 4^2 \times 0.004$$

$$= 172.8\ N.$$

A plate of 600mm length & 400mm wide is immersed in a Fluid of sp. gravity 0.8 & K.V = 10^{-4} m²/s. The Fluid is Moving with a velocity of 6 m/s. Determine Boundary layer Thickness, Drag Force Coefficient & Shear stress.

Given data

length (L) = 600mm = 0.6m

wide (b) = 400mm = 0.4m

Sp. gravity = 0.8 ⟹ 1000 × 0.8 = 800 Kg/m³.

Kinematic viscosity (ν) = 10^{-4} m²/s

Velocity (V) = 6 m/s.

$$Re = \frac{U \cdot L}{\nu} = \frac{6 \times 0.6}{10^{-4}} = 3.6 \times 10^{4}$$ - laminar flow.

$$\delta = \frac{5L}{\sqrt{Re_L}} = \frac{5 \times 0.6}{\sqrt{3.6 \times 10^{4}}} = 0.0158$$

$$C_D = \frac{1.328}{\sqrt{3.6 \times 10^{4}}} = 0.007$$

$$F_D = \frac{1}{2} \times 800 \times 0.24 \times 6^2 \times 0.007 = 24.192$$

Shear stress $\tau_0 = 0.332 \frac{\rho u^2}{\sqrt{Re}}$

$$= \frac{0.332 \times 800 \times 6^2}{\sqrt{3.6 \times 10^{4}}} = 50.39 \text{ N/m}^2.$$

Air is flowing over a flat plate 500 mm long, 600 mm wide with a velocity of 4 m/sec. K.V of Air - 0.15×10^{-4} m²/sec. Determine the boundary layer thickness & shear stress & drag force co-efficient, Density of Air 1.24 Kg/m³.

Sol: Given:-

l = 0.5 m

b = 0.6 m

U = 4 m/s

ν = 0.15×10^{-4} m²/s

ρ = 1.24 Kg/m³

Reynolds Number (Re) = $\frac{UL}{\nu}$

$$= \frac{4 \times 0.5}{0.15 \times 10^{-4}} = 13.33 \times 10^{4}$$

$\therefore$ $Re < 5 \times 10^5$ so it is laminar

Boundary layer thickness:-

$$\delta = \frac{5L}{\sqrt{Re}} = \frac{5 \times 0.5}{\sqrt{13.33 \times 10^4}} = 0.0068$$

Shear Stress:-

$$\tau_0 = 0.332 \frac{\rho U^2}{\sqrt{Re}}$$

$$= 0.332 \times \frac{1.24 \times 16}{\sqrt{13.33 \times 10^4}} = 0.018 \ KN/m^2$$

$$= 18 \ N/m^2.$$

Drag Force:-

$$F_D = 1/2 \ \rho A U^2 \times C_D$$

$$C_D = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{13.33 \times 10^4}} = 0.0036$$

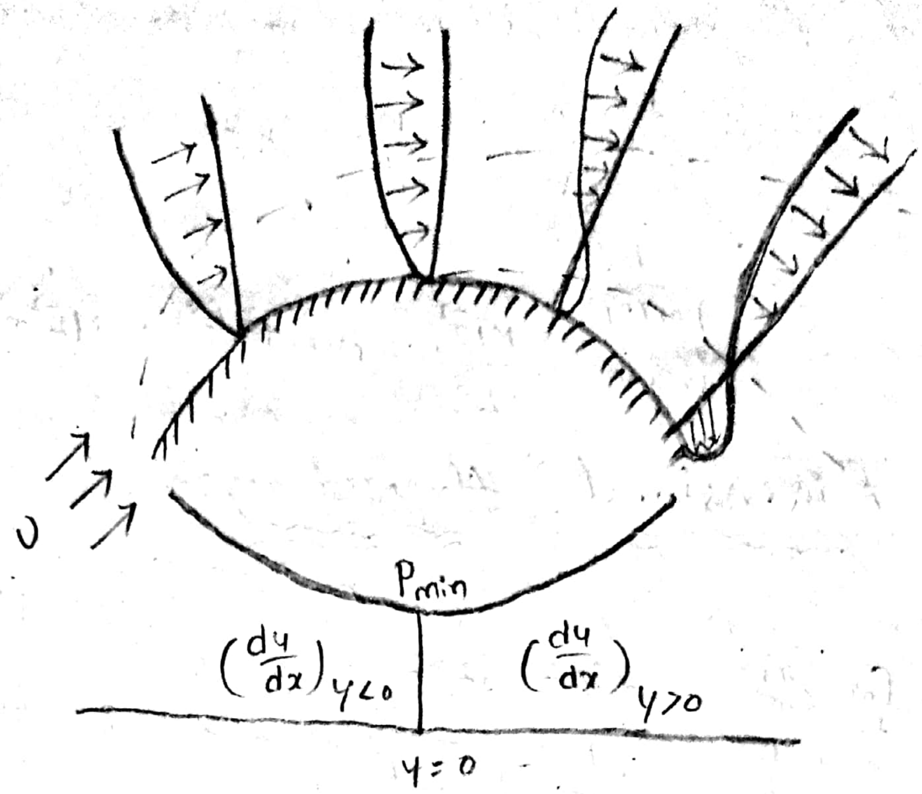
$$C_D = 0.0036$$

$$F_D = 1/2 \times 1.24 \times 0.3 \times 16^2 \times 0.0036$$

$$= 0.0107 m$$

$$F_D = 10.7 mm.$$

3.6.19 **Boundary Separation :-**



Determine the dimensions of quantities.

1. Angular velocity ⇒ $\frac{\text{Angular Covered in radians}}{\text{Time}} = \frac{1}{T} = T^{-1}$
2. Angular Acceleration ⇒ $\text{radian}/\text{sec}^2 = 1/T^2 = T^{-2}$
3. Discharge ⇒ Area x velocity ⇒ $L^2 \times \frac{L}{T} = L^3 T^{-1}$ $\left\{ \because V = \frac{\text{Head}}{\text{time}} \quad V = \frac{L}{T} \right\}$
4. Kinematic viscosity ⇒
5. Force ⇒ $F = m \times A = m \cdot \frac{L}{T^2} \Rightarrow MLT^{-2}$ (mass x acceleration)
6. Spe. weight ⇒
7. Dynamic viscosity ⇒ $\mu = M \cdot L^{-1} T^{-1}$

4. $\nu = \frac{\mu}{\rho}$

Ans: Shear stress $\tau = \mu \cdot \frac{du}{dy}$

$$\mu = \frac{\tau}{du/dy} = \frac{\text{Force/Area}}{1/T} = \frac{\frac{MLT^{-2}}{L^2}}{1/T} = \frac{MLT^{-2}}{L^2 \cdot T^{-1}}$$

$$\mu = M \cdot L \cdot T^{-2} \cdot L^{-2} \cdot T$$

$$\mu = M \cdot L^{-1} \cdot T^{-1}$$

$v = \frac{\mu}{\rho} = \frac{M \cdot L^{-1} \cdot T^{-1}}{ML^{-3}}$

$L^{-1} \cdot T^{-1} L^{3}$

$\rho = \frac{mass}{volume} = \frac{m}{L^3}$ $L^2 T^{-1}$.

6 $w = \frac{Force}{Volume} = \frac{MLT^{-2}}{L^3} = M \cdot L \cdot T^{-2} \cdot L^{3} = ML^{-2}T^{-2}$.

Ans

Dimensional Homogeneous :-

$V = \sqrt{2gh}$

for Exi $V = \sqrt{2gh}$

$\frac{L}{T} = \sqrt{\frac{L}{T^2} \times L}$ → LHS = RHS

Square rule

$\frac{L}{T} = \sqrt{\frac{L^2}{T^2}}$ $\because \sqrt{\frac{L^2}{T^2}} \neq (\frac{L}{T})^2$

$$\frac{L}{T} = \frac{L}{T}$$

- A Time Period of "T" Pendulum Depends on the length "L" of the Pendulum & Acceleration due to gravity "G". Derive the Expression for a Time Period.

Ans

$t = L \cdot g$

$T = K \, L^{a} \cdot g^{b}$

$T^{1} = K \cdot L^{a} \cdot (L \cdot T^{-2})^{b}$

$1 = 1x - 2b$ $0 = a + b$

$1 = -2b$ $a = -b$

$b = -1/2$ $a = -(-1/2)$

$a = 1/2$

$$T = K \cdot L^{1/2} \cdot (L \cdot T^{-2})^{-1/2}$$

$$= K \cdot L^{1/2} \cdot g^{-1/2}$$

$$T = K \cdot \frac{L^{1/2}}{g^{1/2}} \Rightarrow \boxed{K \cdot \sqrt{\frac{L}{g}}}$$

6/8/16

Find the Expression for drag Force on smooth sphere dia. 'd' Moving with a uniform velocity 'v' in a Fluid of density 'ρ' & Dynamic viscosity μ.

Sol:

$$F = K \cdot D^a \, v^b \, \rho^c \, \mu^d$$

$$m^1 L^1 T^{-2} = K \cdot L^a (LT^{-1})^b (mL^{-3})^c (mL^{-1}T^{-1})^d$$

$$m \quad 1 = c + d$$
$$L \quad 1 = a + b - 3c - d$$
$$T \quad -2 = -b - d$$

$$\left\{ c = 1 - d, \quad b = 2 - d \right\}$$

$$a = 1 - b + 3c + d$$
$$= 1 - (2-d) + 3(1-d) + d$$
$$= 2 - d$$

$$mLT^{-2} \cdot L^{2-d} (LT^{-1})^{2-d} (mL^{-3})^{1-d} (mL^{-1}T^{-1})^d$$

$$F = K \cdot D^{2-d} \cdot V^{2-d} \cdot \rho^{1-d} \cdot \mu^d$$

$$= K \cdot D^2 V^2 \rho \, (D^{-d} V^{-d} \rho^{-d} \mu^d)$$

$$F = K \cdot D^2 \cdot V^2 \rho \left(\frac{\mu}{DV\rho} \right)^d$$

The Resistance force "R" of a Supersonic plane during flight can be considered as depends up on the length of the Air gap "L". velocity "V". Air viscosity "μ" & Air density "ρ" & Bulk modulus of Air "K". Derive an Expression for the Functional Relationship between these variables and Resisting Force.

Sol: **Given :-**

$$R = A\, L^a \cdot V^b \cdot \mu^c \cdot \rho^d \cdot K^e$$

$$MLT^{-2} = A \cdot L^a (LT^{-1})^b \cdot (ML^{-1}T^{-1})^c \cdot (ML^{-3})^d \cdot (M \cdot L^{-1} \cdot T^{-2})^e$$

$M \to 1 = c+d+e \Rightarrow \therefore d = 1-c-e$

$L \to 1 = a+b-c-3d-e \Rightarrow a = 1-b+c+3d+e$

$T \to -2 = -b-c-2e \Rightarrow b = 2-c-2e.$

$a = 1-b+c+3d+e.$ — These values are Substitute in Equation a.

$= 1-(2-c-2e)+c+3(1-c-e)+e$

$= 1-2+c+2e+c+3-3c-3e+e$ — $2c-3c = -c$

$= 1-2+2c-3c+2e-3e+e$

$= -1-1c+0$

$= 2-c$

$$R = A \cdot L^{2-c}\, V^{2-c-2e} \cdot \mu^c \cdot \rho^{1-c-e} \cdot K^e$$

$$= A \cdot L^2\, V^{2-2e} \cdot \rho^{1-e}\, K^e (L^{-c}\, V^{-c}\, \mu^c\, \rho^{-c})$$

$$= A \cdot L^2 V^2 V^{-2e} \cdot \rho \cdot \rho^{-e} \cdot K^e (L^{-c}\, V^{-c}\, \mu^c\, \rho^{-c})$$

$$= A \cdot L^2 \cdot V^2 \cdot \rho \left(V^{-2e} \rho^{-e} K^e \right) \left(\frac{\mu}{LV\rho} \right)^c$$

$$= A \cdot L^2 \cdot V^2 \cdot \rho \left(\frac{K}{V^2 \rho} \right)^e \cdot \left(\frac{\mu}{LV\rho} \right)^c$$

Unit-III: Boundary layer theory

FLUID MECHANICS

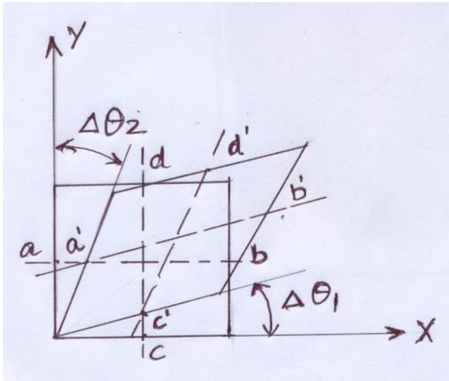
FLOW PAST IMMERSED BODIES

Whenever a body is placed in a stream, forces are exerted on the body. Similarly, if the body is moving in a stationary fluid, force is exerted on the body.

Therefore, when there is a relative motion between the body and the fluid, force is exerted on the body.
Example: Wind forces on buildings, bridges etc., Force experienced by automobiles, aircraft, propeller etc.,

FORCE EXERTED BY FLOWING FLUID ON A STATIONARY BODY

Consider a stationary body placed in a stream of real fluid.
Let U = Free stream velocity.
Fluid will exert a Force F R on the body.
The force is inclined at an angle to the direction of velocity.
The Force F R can be resolved into TWO components – One in the direction of flow (F F D. and the other perpendicular to it (FL).
R = FL + FD
Drag: The component of the total force (F Drag (F R. in the direction of motion is called as



). Drag is the force exerted by the fluid on the body in the direction of motion. Drag resists motion of the body or fluid.
Example: Wind resistance to a moving car, water resistance to torpedoes etc., Power is required to overcome drag and hence drag has to be reduced to a possible
minimum.

Lift: The component of the total force in the direction perpendicular to the direction of motion. Lift is the force exerted by the fluid normal to the direction of motion.
Lift is zero for symmetrical flow.
Lift = Weight (in the case of an airplane in cruise)
Consider an elemental area (dA) on the surface of the body.
1. Pressure force (PdA) acts normal to the area dA.
2. 0dA) acts along the tangent to dA
Shear force (

□
3. (□) = Angle made by force pdA with horizontal.
dF D = Drag force on the element
= (pdA)Cos (□) + (□
0dA) Sin (θ)
Therefore, Total drag on the body
= F
D = _dFD = _(pdA)Cos (θ) + _(τ
0dA) Sin (θ) -----Equation (1)
Total drag (or Profile drag) = Pressure drag (or form drag) + Friction drag.

The quantity _(pdA) Cos (□) is called the pressure drag or form drag and depends upon the form or shape of the body as well as the location of the separation point.
The quantity 0dA) Sin (θ) is called as the friction drag or skin friction drag and
_(
□

depends upon the extent and character of the boundary layer. The sum of the pressure drag and the friction drag is called as total drag or profile drag.

In the case of a flat plate (Fig. a), (□) = 900. Hence,D is only the friction drag
F
If the plate is held normal to the plane (Fig. b), (□) = 00, Hence FD is only the pressure drag
Lift = Force due to Pressure in the normal direction + Force due to shear in the normal direction.
= – (pdA)Sin (□) +0dA) Cos (θ) OR FL = – _(pdA)Sin (θ) + _(τ0dA) Cos (θ)
(□ -----Equation (2)

Equations (1) and (2) require detailed information regarding pressure distributions and shear stress distributions to determine F D and FL on the body.

As a simple alternative, Drag and Lift Forces are expressed as

F D = CDA (ρU2/2)

F L = CLA (ρU2/2)

Where C D and CL are called Coefficient of Drag and Coefficient of Lift respectively,

? = Density of fluid, U = Velocity of body relative to fluid

A = Reference area or projected area of the body perpendicular to the direction of flow or it is the largest projected area in the in the case of submerged body.

(□U2/2) = Dynamic pressure.

GENERAL EQUATIONS FOR DRAG AND LIFT

Let force 'F' is exerted by fluid on the body.
F = F(L, , , k, U, g) where L = Length, = Density, = Viscosity, k = Bulk modulus of elasticity, U = Velocity and g = Acceleration due to gravity. From dimensional analysis, we get,

F = L2U2 f(Re, Fr, M)
Where Re = Reynolds Number = (UL/ m),
Fr = Froude Number = (U/√gL)
M = Mach Number = (U/√k/) = (U/a); a = Sonic velocity
If the body is completely submerged, Fr is not important. If Mach number is relatively low (say, < 0.25), M can be neglected.
Then, F = L2U2 f(Re) or
F D = CD L2 (ρU2/2) = CD Area (ρU2/2)
F L = CL L2 (ρU2/2) = CL
C L and CD are the coefficients of Lift and Drag respectively

TYPES OF DRAG

The type of drag experienced by the body depends upon the nature of fluid and the shape of the body:

1. Skin friction drag
2. Pressure drag
3. Profile drag
4. Wave drag
5. Induced drag

Skin Friction Drag: The part of the total drag that is due to the tangential shear stress (0) acting on the surface of the body is called the skin friction drag. It is also called as friction drag or shear drag or viscous drag.

Pressure Drag: The part of the total drag that is due to pressure on the body is called as Pressure Drag. It is also called as Form Drag since it mainly depends on the shape or form of the body

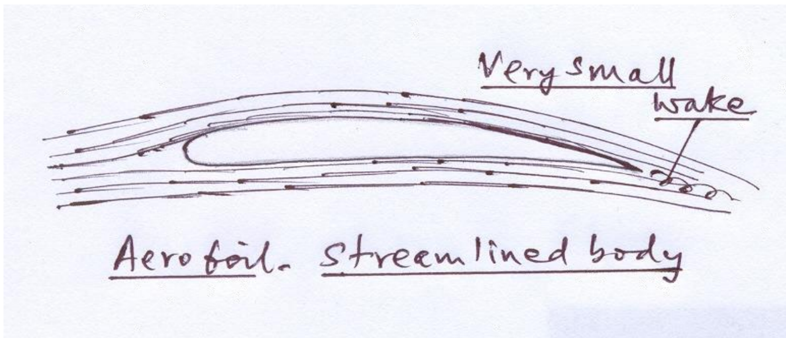


Fig. Flow over Bodies – Pressure Drag and Skin Friction Drag

For a streamlined body, pressure drag is small. Large part of drag is due to friction. Ex., Aerofoils, modern cars etc., - Streamlines match with the surface and there is very small wake behind the body.
For a bluff body, streamlines don't match with the surface. Flow separates and gives rise to large wake zone. Pressure drag is predominant compared to friction drag – Ex., Bus body.

5

Profile Drag or Total Drag is the sum of Pressure or Form drag and Skin Friction drag.

Wave Drag: When a body like ship moves through a fluid, waves are produced on the surface of the liquid. The drag caused due to these waves is called as wave drag. The wave drag is obtained by subtracting all other drags from the total drag measurements. The drag, which is caused by change in pressure due to a shock wave in supersonic flow, is also called as wave drag.

Induced Drag: When a body has a finite length (Ex., Wing of an airplane), the pattern of flow is affected due to the conditions of flow at the ends. The flow cannot be treated as two-dimensional, but has to be treated as three-dimensional flow. Due to this, body is subjected to additional drag. This drag, due to the three dimensional nature of flow and finite length of the body is called as Induced Drag.

Deformation Drag: If the body with a very small length (Ex., Sphere) moves at very low velocity through a fluid with high kinematics viscosity (Re = ($\rho UL/\mu$) less than 0.1), the body experiences a resistance to its motion due to the wide spread deformation of fluid particles. This drag is known as Deformation Drag.

Problem –1.

A circular disc 3m in diameter is held normal to 26.4m/s wind velocity. What force is required to hold it at rest? Assume density of air = 1.2kg/m3, and C_D = 1.1.

Force required to hold the disc = Drag = FD =CDA(ρU2/2)
= 1.1□ (□□32/4) □ (1.2 □26.42/2) = 3251.5 N

Problem-2.

Calculate the power required to overcome the aerodynamic drag for the two cars both traveling at 90km/h using the following data.

Car (A) – CD = 0.8, A (frontal) = 2m2,

Car (B) – CD = 0.4, A (frontal) = 1.8m2. Take ρ = 1.164 kg/m3.

For Car (A)

Power = Force □ Velocity = F UD. U

= 90km/hr = 25m/s.

Power = CD A(ρU2/2) ×U

=0.8□ 2□(1.164□252/2) □25 = 14550W = 14.55kW Similarly for Car (B),

Power = 0.4□ 1.8 □ (1.164 □252/2) □25 = 6.55kW

Problem-3.

Experiments were conducted in a wind tunnel with a wind speed of 50km/h. on a flat plate of size 2m long and 1m wide. The plate is kept at such an angle that the co-efficient of lift and drag are 0.75 and 0.15 respectively. Determine (a) Lift force

(b) Drag force (c) Resultant force (d) Power required to maintain flow.

Take □ = 1.2 kg/m3.

Given: A = 2m2; CL = 0.75; CD = 0.15; ρ = 1.2 kg/m3; U= 13.89m/s

Drag force = F LiftD = CDA(ρU2/2) = 34.72N

force = F

L = CLA(ρU2/2) = 173.6N

Resultant force = FR = (FD2 + FL2)1/2 = 177.03 N

Power = F

D □□□□□□□□□□□□□□□□□

BOUNDARY LAYER CONCEPT

Ideal fluid theory assumes that fluid is ideal, zero viscosity and constant density. Results obtained don't match with experiments.

With ideal fluid, there is no drag force. However, in practice, drag force exists. In practice, fluids adhere to the boundary.

At wall, fluid velocity = wall velocity- this is called No Slip Condition. The velocity of the fluid is zero at the wall and goes on increasing as we go away from the wall if the wall is stationary.

This variation in velocity near the wall gives rise to shear stresses resulting in resistance to motion of bodies.

CONCEPT OF BOUNDARY LAYER

L.Prandtl developed Boundary Layer Theory
Boundary layer theory explains the drag force experienced by the body. The fluid in the vicinity of the surface of the body may be divided into two regions – (1) Boundary layer and (2) Potential flow or Ir-rotational flow region.

7

BOUNDARY LAYER

Boundary layer is a very thin layer of fluid in the immediate vicinity of the wall (or boundary). When a real fluid flows past a solid boundary, there develops a thin layer very close to the boundary in which the velocity rapidly increases from zero at the boundary (due to no slip condition) to the nearly uniform velocity in the free stream. This region is called Boundary layer. In this region, the effect of viscosity is predominant due to the high values of (du/dy) and most of the energy is lost in this zone due to viscous shear.

The layer of fluid which has its velocity affected by the boundary shear is called as Boundary Layer. A thin layer of fluid in the vicinity of the boundary, whose velocity is affected due to viscous shear, is called as the Boundary layer

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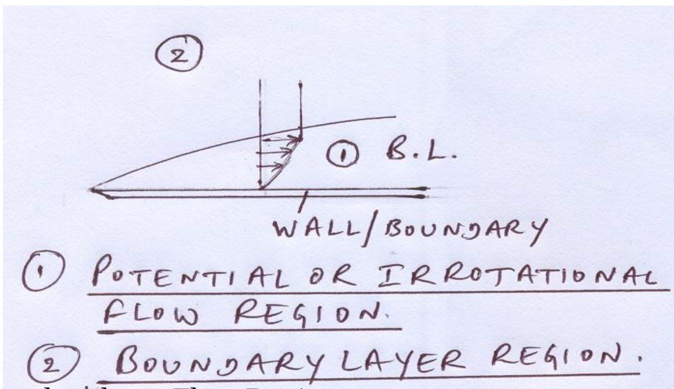


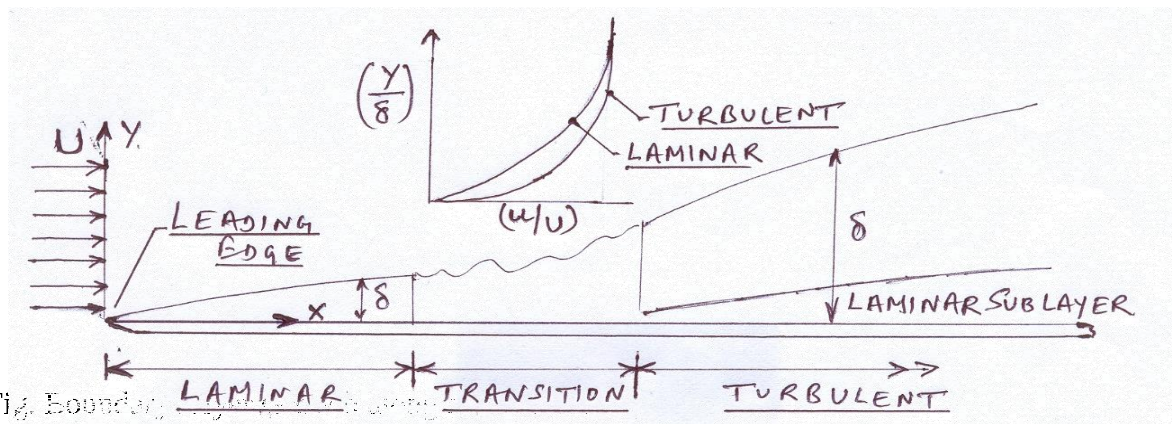
Fig. Potential and Boundary layer Flow Regions

POTENTIAL FLOW OR IRROTATIONAL FLOW REGION

The portion of the fluid outside the boundary layer where viscous effects are negligible is called potential flow or ir-rotational flow region. The flow in this region
Can be treated as Ideal Fluid Flow.

BOUNDARY LAYER ALONG A FLAT PLATE AND IT'S uCHARECTERISTICS

Consider a steady, uniform stream of fluid moving with velocity (U) on a flat plate. Let U = Free stream velocity or Ambient velocity. At the leading edge, the thickness of the boundary layer is zero. In the down stream direction, the thickness
of the boundary layer (□) goes on increasing as shown.



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Up to a certain length along the plate from the leading edge, boundary layer thickness increases and the boundary layer exhibits the characteristics of a laminar flow irrespective of whether the incoming flow is laminar or turbulent. – This is known as laminar boundary layer.
The thickness of the laminar boundary layer (□) is given by (□) = y (at (u/U) = 0.99)
where u = local velocity.

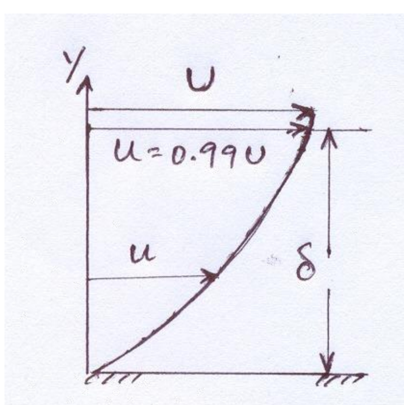
The thickness of the laminar boundary layer is given by (□) = [5x/(R ex)0.5]
Where Rex = Reynolds number based on distance from the leading edge (x)
Rex = (Ux/ν); Therefore, (δ) = 5(xν/U)0.5
In the laminar boundary layer, the Newton's law of viscosity (□) = μ (du/dy) is valid
and the velocity distribution is parabolic in nature.

Beyond some distance from the leading edge, the laminar boundary layer becomes unstable and the flow in the boundary layer exhibits the characteristics between laminar and turbulent flows. This region is known as the transition region. After this region, the thickness of the boundary layer increases rapidly and the flow in the boundary layer exhibits the characteristics of the turbulent flow
This region is known as the turbulent boundary layer. In the turbulent boundary layer, the boundary layer thickness is given by
(□) = [0.377x/(R ex)0.2]
The velocity profile is logarithmic in the turbulent boundary layer.

The change from laminar to turbulent boundary layer depends mainly on R ex =(Ux/ □). The value of critical Reynolds number varies from 3 □105 to 6□105 (for a flat plate). For all practical purposes, we can take R

x. Distance from the leading edge.

r = 5×105



If the plate is smooth, the turbulent boundary layer consists of a thin layer adjacent to the boundary in which the flow is laminar. This thin layer is known as the laminar sub-layer.
The thickness of the laminar sub-layer (□’) is given by
he laminar sub-layer, although very thin is an important factor in deciding whether a surface is hydro-dynamically smooth or rough surface.

FACTORS AFFECTING THE GROWTH OF BOUNDARY LAYERS

1. Distance (x) from the leading edge – Boundary layer thickness varies directly with the distance (x). More the distance (x), more is the thickness of the boundary layer.
2. Free stream velocity – Boundary layer thickness varies inversely as free stream velocity.
3. Viscosity of the fluid – Boundary layer thickness varies directly as viscosity.
4. Density of the fluid – Boundary layer thickness varies inversely as density.

THICKNESSES OF THE BOUNDARY LAYER

Boundary layer thickness - It is the distance from the boundary in which the local velocity reaches 99% of the main stream velocity and is denoted by (□).
y = (□) when u=0.99U

Displacement Thickness (□□□□ *): It is defined as the distance perpendicular to the boundary by which the boundary will have to be displaced outward so that the actual discharge would be same as that of the ideal fluid past the displaced boundary. It is also defined as the distance measured perpendicular from the actual boundary such that the mass flux through this distance is equal to the deficit of mass flux due to boundary layer formation.

Deficit of mass flow (discharge) = (b.dy)(U-u)□
Total deficit of mass flow:
0 _∝ ρ(b.dy)(U-u) = ρbδ*U
δ * =
0
_δ (1- u/U)dy

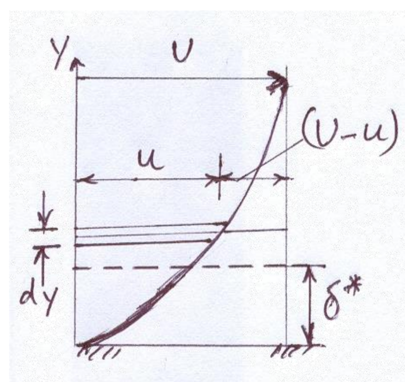
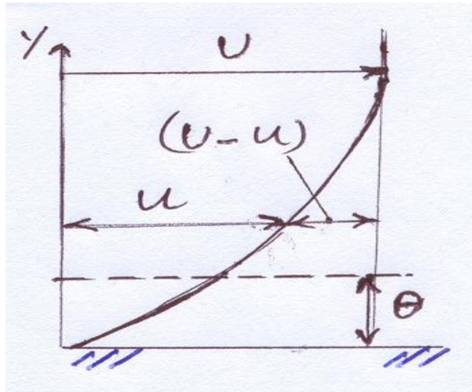


Fig. Displacement Thickness

Momentum thickness (□ ?): It is defined as the distance measured
□□
perpendicular from the actual boundary such that the momentum flux through this distance is equal to the deficit of the momentum flux due to the boundary layer formation.
Momentum deficit = □(b.dy)(U -u)u
Total momentum deficit = Moment through thickness (□)
0 _∝ ρ (b.dy)(U-u)u = ρbθU2
(□) = 0_δ (1- u/U)(u/U)dy


y
U
(U-u)
u
θ